All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator **is** *permitted in this examination.*

Throughout all rings are assumed to be commutative with a 1, and all free modules are assumed to be of finite rank.

1. (a) A quadratic $f(x)$ is given by:

$$f(x) = x^2 - x - 12.$$

Express the above quadratic in factorised form and hence write down the roots of the quadratic.

(b) Express the quadratic in the following form:

$$f(x) = (x - q)^2 + r$$

and hence write down the minimum value of the function and the value of x at which the minimum occurs.

(c) Sketch the curve $y = f(x)$, clearly specifying the co-ordinates at which the curve crosses the x and y axis.

(d) Find the roots of the following equations:

$$\begin{aligned} i) \quad f(x) &= e^{2x} + e^x - 2 \\ ii) \quad f(x) &= 2^{2x+1} + 2^{\log_2 3 + x} - 2 \end{aligned}$$

2. (a) Factorise the following

$$x^3 - 6x^2 + 11x - 6 = 0.$$

(b) Find all the stationary point to the function:

$$f(x) = x^3 - 6x^2 + 11x - 6$$

and state whether each point is a local maximum or a local minimum. Give reasons for your answers.

(c) Sketch the curve $y = f(x)$ noting all the points where the curve crosses the x and y axis

3. (a) By considering a small $\delta > 0$ and writing $x = 1 + \delta$ and $x = 1 - \delta$, compute the upper and lower limits of the function:

$$f(x) = \begin{cases} 2x + a & \text{if } -\infty < x \leqslant 1, \\ x^2 - \frac{1}{2}ax - 1 & \text{if } 1 < x < \infty \end{cases}$$

(b) What value of a (if any) will make $f(x)$ a continuous?

(c) Compute the following limits:

$$\begin{aligned} l_1 &= \lim_{x \to 0} \frac{e^x \sin(\pi x) - x}{2x \cos x - \tan x} \\ l_2 &= \lim_{x \to \infty} \frac{x^2 - 2x + 1}{\pi x^2 - 3x + 2} \end{aligned}$$

4. (a) Solve the following simultaneous equations:

$$\begin{aligned} 2x - 4y &= -6 \\ x + y &= 3 \end{aligned}$$

Plot the linear functions $y(x)$ specified by each of the equations above and show the points at which they intersect the x and y axis, and where they intersect each other.

(b) Construct the line passing through the points $(-1, 0)$ and $(1, 1)$ and write it in the form:

$$ax + by = c$$

(c) Calculate the shortest distance between the line

$$4x + 3y = 25$$

and the point $(0, 0)$

5. (a) Find all the solutions to the following trigonometric equation:

$$\sin 2\theta = \frac{1}{2} \quad 0 \leqslant \theta \leqslant 2\pi$$

Hint: You may use the fact that $\sin(\pi - \theta) = \sin\theta$

(b) Find all solutions to the equation:

$$4\sin^2\theta - 1 = 0 \quad 0 \leqslant \theta \leqslant 2\pi$$

(c) Prove the following trigonometric identities:

$$\begin{array}{ll} i) & 1+\cot^2\theta \equiv \text{cosec}^2\theta \\ ii) & \frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta} \equiv \frac{4}{\sin^2 2\theta} \end{array}$$

6. Differentiate the following function with respect to x

   (a) $f(x) = x\sin x$

   (b) $f(x) = e^{e^x}$

   (c) $f(x) = \frac{x\sin x}{1+x^2}$

   (d) Compute the first three terms in the Maclaurin expansion of the following function:

$$f(x) = \frac{1}{1-x}$$

7. (a) Write the following complex numbers in the form $z = a + bi$

$$\begin{aligned} z_1 &= (1+2i)-(1-2i) \\ z_2 &= (1+2i)(1-2i) \\ z_3 &= \frac{1+2i}{1-2i} \end{aligned}$$

   (b) Sketch the above on an Argand diagram.

   (c) Write the complex number

$$z = \frac{\sqrt{3}}{2}+\frac{i}{2}$$

   in the forms:

$$z = re^{i\theta} \quad \text{and} \quad z = r(\cos\theta + i\sin\theta)$$

   (d) Compute z^2 and z^3 using either the polar or exponential form of z