Elementary Mathematics 6101

Exercise Sheet 2 Solutions

September 9, 2011

1. (a) at the points $x = 2, -2$ the expression will be become undefined but will be fine for all other values, so the domain is $\mathbb{R}/\{-2,2\}$

   (b) The domain is all of $\mathbb{R}$.

   (c) For $f(x_{=}\sqrt{x^2-4}$, we know that you can't take the square root of a negative number and so $x^2 - 4 > 0 \Rightarrow |x| > 2$ So the domain is $-\infty \leqslant x \leqslant -2$ and $2 \leqslant x \leqslant \infty$

   (d) Taking cube roots of negative numbers is fine, so the domain is all of $\mathbb{R}$

2. (a) $f \circ g(x) = g(f(x)) = (x^2 - 1)^{-1}$

   (b) $g \circ f(x) = f(g(x)) = (x - 1)^{-2}$

   (c) $g \circ h(x) = h(g(x)) = \sqrt{(x-1)^{-1} + 1} = \sqrt{\frac{x}{x-1}}$

   (d) $h \circ f(x) = f(h(x)) = (\sqrt{x+1})^2 = x + 1$

3. We have $f \circ h(x) = h(f(x)) = \sqrt{x+1} = \sqrt{f(x)}$ and so replacing $f(x)$ by X say, we have $h(X) = \sqrt{X}$, so we can say that $h(x) = \sqrt{x}$

4. (a) To find the inverse write: $x = y^3 + 1$, we re-arrange the equation to find y, so $y^3 = x - 1$ which in turn shows that $y = (x-1)^{\frac{1}{3}}$, so $f^{-1}(x) = (x-1)^{\frac{1}{3}}$

   (b) $x = y - 4 \Rightarrow y = x + 4$, so $g^{-1}(x) = x + 4$

   (c) $x = 1 - \frac{1}{y} \Rightarrow \frac{1}{y} = 1 - x \Rightarrow y = (1-x)^{-1}$ and so $h^{-1}(x) = (1-x)^{-1}$