# Exercise Sheet 3 - Solutions

September 11, 2011

1. (a) $m = \frac{6-2}{4-1} = \frac{4}{3}$
   (b) $m = \frac{2-0}{-1-1} = -\frac{2}{2} = -1$
   (c) $m = \frac{0-3}{0-2} = \frac{-3}{-2} = \frac{3}{2}$
2. (a) $\frac{4}{3} = \frac{6-y}{4-x} \Rightarrow 4(4-x) = 3(6-y) \Rightarrow 3y - 4x - 2 = 0$
   (b) $-1 = \frac{2-y}{-1-x} \Rightarrow -1(-1-x) = 2-y \Rightarrow 1+x = 2-y \Rightarrow y+x-1 = 0$
   (c) $\frac{3}{2} = \frac{0-y}{0-x} \Rightarrow 2y - 3x = 0$
3. The equation of the line can be written as $y = -4x/3-2/3$, the gradient of the normal is then $n = 3/4$, the general equation can be written as $y = 3x/4 + C$, using the point at which the line passes $(1, 1)$ shows that: $1 = 3/4 + C$ which shows that $C = 1/4$ and the equation of the normal is $4y = 3x + 1$.
4. (a) Add the two equations to obtain $2x + x + y - y = 2 + 1 \Rightarrow 3x = 3 \Rightarrow x = 1$ and so inserting this into either of the equations shows that $y = 1$
   (b) Add the two equations to obtain $2y + y + x - x = 1 \Rightarrow 3y = 1 \Rightarrow y = 1/3$, inserting this into any of the equations shows that $x = -5/3$
5. The equation of the line can be written as $y = 2x - 1$ and so we can read of the gradient of the normal easily as $n = -1/2$.

   (a) The equation for the normal is written as $y = -x/2 + C$, the normal goes through the point $(2, 1)$, so inserting these values into the equation for the normal shows that $1 = -2/2+C \Rightarrow C = 2$ So the equation for the normal is $2y+x = 4$. The point of intersection is found by simultaneously solving the equation for the normal and the line so we solve the pair of equations $y = 2x - 1$ and $2y + x = 4$ Multiplying the first equation by 2 and subtracting the second equation shows that $2y - 4x - (2y + x) = -2 - 4 \Rightarrow -5x = -6 \Rightarrow x = 6/5$, inserting this values into either of the other equations shows that $y = 7/5$, so the distance required can just be used $\ell^2 = (2-6/5)^2 + (1-7/5)^2 = 4/5$ and so the shortest distance from the point to the line is $\ell = 2/\sqrt{5}$

(b) The equation for the normal is written as $y = -x/2 + C$, the normal goes through the point $(0, 0)$, so inserting these values into the equation for the normal shows that $C = 0$ and the equation of the normal is $2y + x = 0$. The task now is to compute the intersection of these two multiplying the equation for the line by two and subtracting the equation for the normal to obtain $2y - 4x - (2y + x) = -2 \Rightarrow -5x = -2 \Rightarrow x = 2/5$, inserting this value into any of the other equations shows that $y = -1/5$, The distance between $(0, 0)$ and the point of intersection is $\ell^2 = (0 - 2/5)^2 + (0 - (-1/5))^2 = (2/5)^2 + (1/5)^2 = 4/25 + 1/25 = 1/5$ and hence $\ell = 1/\sqrt{5}$