

# Elementary Mathematics 6101

## Exercise Sheet 6

17 November 2010

Please answer all the questions on the sheet

1. Differentiate from first principles

a)  $y = (x + 1)^2$

b)  $y = (3x - 4)(x + 4)$

2. Differentiate the following using any rules that you may know of.

a)  $y = x^{-3} + 2x - 2$

b)  $y = \left(\frac{1}{x}\right)(1 + x^2)$

c)  $y = \frac{\sqrt{x+7}}{x^2}$

3. Suppose that there are two functions  $f(x)$  and  $g(x)$  whose derivative satisfy  $f'(x) = g(x)$  and  $g'(x) = -f(x)$

a) By use of the chain rule show that:

$$(h^2)'(x) = 2h(x)h'(x)$$

b)  $(f^2(x) + g^2(x))'(x) = 0$  for all values of  $x$

4. Compute the following limits by use of L'Hopital's rule

a)  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 + x^2}}{x}$

b)  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{6x^3 - 7x^2 + 1}$

5. Using Maclaurin's series, find the expansions of the following functions:

a)  $f(x) = \frac{1}{\sqrt{1+x}}$ , the first three terms

b)  $g(x) = \frac{1}{1+\sqrt{1+x}}$ , the first two terms and if you are feeling strong the third term.

6. Find the turning points of the following functions and classify each one

a)  $y = 4x^3 + 3x^2 - 6x - 1$

b)  $y = \frac{9}{x} + x$

7. A farmer has an  $80m$  length of fencing. He wants to use it to form three sides of a rectangular enclosure against an existing wall which will be the fourth side.

a) Denote the length of one of the sides by  $x$ , what is the area of the enclosure?

b) By use of differentiation or otherwise find the maximum area of the enclosure.