Elementary Mathematics 6101

Exercise Sheet 6

24 November 2010

Please answer all the questions on the sheet

1. Differentiate the following functions
   a) $f(x) = \tan x$
   b) $f(x) = \sin^2 x$
2. Compute the following limits
   a) $\lim_{x \to 0} \dfrac{\tan x - x}{\sin x - x}$
   b) $\lim_{x \to 0} \dfrac{x \cos x - \sin x}{x^3}$
3. Solve the following equations:
   a) $\sin^2 2\theta = \frac{1}{2} \quad 0 \leqslant \theta \leqslant \pi$
   b) $1 - 2\sin^2 2\theta + 2\cos^2 2\theta = 0 \quad 0 \leqslant \theta \leqslant \pi$
4. Simplify
$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
5. Prove that
$$(\operatorname{cosec} x - \sin x)(\sec x - \cos x) = \frac{1}{\tan x + \cot x}$$