

# Elementary Mathematics 6101

## Exercise Sheet 8

8 December 2010

Please answer all the questions on the sheet

1. Solve the following and put the solution in the form of  $-1 + \log_5 k$

$$2 \cdot 5^{x+1} = 1 + \frac{3}{5^x}$$

2. Write  $x^n = e^{\ln x^n} = e^{n \ln x}$  and use the fact proved in class that if  $f(x) = e^{g(x)}$  then  $f'(x) = e^{g(x)}g'(x)$  to show that in general the derivative of  $x^n$  is  $nx^{n-1}$ .
3.  $f(x) = g \circ h(x)$  where  $h(x) = \ln x$ , so  $f(x) = \ln(f(x))$ , find  $f'(x)$ .
4. By using L'Hopital's rule and also by series substitution, find the values of the following limits:

a)  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2 \sin x}{x^3}$  (Apply L'Hopital's rule more than once.)

b)  $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

5. Differentiate the following functions:

a)  $f(x) = 2^x \ln x$

b)  $g(x) = \frac{e^x}{x + \ln x}$

6. Find the first three terms in the Maclaurin series of the function  $f(x) = 2^x$