Exercise Sheet 4- Solutions

September 12, 2011

1. (a) $2x^2 - 3x + 1 = (2x - 1)(x - 1) = 0$ which shows that $x = 1$ and $x = 1/2$

   (b) $x^2 - 4x + 2 = (x - 2)^2 - 4 + 2 = 0 \Rightarrow (x - 2)^2 = 2 \Rightarrow x = 2 \pm \sqrt{2}$, so the two solutions are $x = 2 + \sqrt{2}$ and $x = 2 - \sqrt{2}$

   (c) $x^2 - 2x + 2 = (x - 1)^2 - 1 + 2 = (x - 1)^2 + 1 = 0$, there are no solutions to this equation.

   (d) $x^2 - 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, there are two solution $x = 3$ and $x = -3$

2. (a) To compute the points of intersection the values of y must be the same, so we equate the y values: $x^2 - 2x + 3 = x^2 - 3x - 1 \Rightarrow x = -4$ and then insert this into one of the quadratics $y = (-4)^2 - 2(-4) + 3 = 16 + 8 + 3 = 27$

   (b) To compute the points of intersection the values of y must be the same, so we equate the y values: $x^2 + 3x - 3 = x - 2 \Rightarrow x^2 + 2x - 1 = (x + 1)^2 - 2 = 0$ so $x = -1 + \sqrt{2}$ and $-1 - \sqrt{2}$. inserting these point into the equation of the line, the points of intersections are $(-1 + \sqrt{2}, -3 + \sqrt{2})$ and $(-1 - \sqrt{2}, -3 - \sqrt{2})$

3. Taking the hint $y = \sqrt{x}$ then $y^2 = x$ ans so the equation becomes $y^2 - 3y - 1 = 0$, completing the square shows $(y - 3/2)^2 - 9/4 - 1 = (y - 3/2)^2 - 13/4 = 0$ which shows that $y = 3/2 \pm \sqrt{13}/2 = \sqrt{x}$. Upon squaring this shows that $x = 6 \pm \sqrt{13}/2$.

4. (a) $x^3 - a^3 = (x - a)(x^2 + a_1 x + a_2) = x^3 + (a_1 - a)x^2 + (a_2 - aa_1)x - aa_2$. Comparing coefficients shows that $a_1 = a$ and $-aa_2 = a^2 \Rightarrow a_2 = a^2$ and so $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

   (b) We are told that $x = -1$ is a solution to the cubic and so $(x + 1)$ is a factor of the cubic $x^3 + 6x^2 + 11x + 6 = (x + 1)(x^2 + a_1 x + a_2) =$

$x^3 + (1 + a_1)x^2 + (a_1 + a_2)x + a_2$ Equating coefficients shows that $a_1 + 1 = 6$ and $a_1 + a_2 = 11$ which makes $a_1 = 5$ and $a_2 = 6$ so $x^3 + 6x^2 + 11x + 6 = (x + 1)(x^2 + 5x + 6) = (x + 1)(x + 2)(x + 3)$

5. From the notes, the equation is $(x - 2)^2 + (y + 1)^2 = 4$

6. Divide the equation of the circle by 4 to obtain $x^2+y^2-x+2y-7/4 = 0$ and then factorise to obtain $(x - 1/2)^2 + 1/4 + (y + 1)^2 - 1 - 7/4 = 0 \Rightarrow (x - 1/2)^2 + (y + 1)^2 = 5/2$, so the circle's centre is at $(1/2, -1)$ with radius $\sqrt{5/2}$