Exercise Sheet 5 - Solutions

September 15, 2011

1. (a) $a_n \to 0$ as $n \to \infty$
   (b) b_n has no limit as $n \to \infty$
   (c) $c_n = \frac{1+2n}{n^2-\sqrt{2}} = \frac{n^{-2}+2n^{-1}}{1-\sqrt{2}n^{-2}} \to \frac{0+0}{1-0} = \frac{0}{1} = 0$
   (d) $d_n = \left(1+\frac{1}{n}\right)^{n/2} = \left[\left(1+\frac{1}{n}\right)^n\right]^{\frac{1}{2}} \to (e)^{\frac{1}{2}} = \sqrt{e}$
2. (a) $\lim_{x\to 1} \frac{2x}{x+1} = \frac{2\times 1}{1+1} = \frac{2}{2} = 1$
   (b) $\lim_{x\to -2} \frac{x+2}{x-4} = \frac{-2+2}{-2-4} = \frac{0}{-6} = 0$
   (c) $\lim_{x\to -1} \frac{3x^2+x-2}{x+1} = \lim_{x\to -1} \frac{(3x-2)(x+1)}{x+1} = \lim_{x\to -1} \frac{(3x-2)\cancel{(x+1)}}{\cancel{x+1}}$ and so $\lim_{x\to -1} 3x-2 = -5$
3. Let us first examine the lower limit when x increases *to* 1, so the idea here is to compute the limit of $f(1-\delta)$ as $\delta \to 0$, as $1-\delta < 1$, we must use the appropriate part of the function. $\lim_{\delta\to 0}(1-\delta)^3 - 2(1-\delta) = \lim_{\delta\to 0} 1 - 3\delta + 3\delta^2 - \delta^3 - 2 + 2\delta = 1 - 2 = -1$ To compute the upper limit as x decreases to 1 and so we compute the limit of $f(1+\delta)$ as $\delta \to 0$. $\lim_{\delta\to 0}(1+\delta)^2 - 2a(1+\delta) + 2 = \lim_{\delta\to 0} 1 + 2\delta + \delta^2 - 2a - 2a\delta + 2 = 1 - 2a + 2 = 3 - 2a$. The function is *continuous* when the upper limit and lower limit are the same. and so we set $3 - 2a = -1$ and find that for the two functions to be continuous at $x = 1$ then $a = 2$