My PhD work involved the examination of free surface flows in electrohydrodynamics. The type of fluids I was considering were irrotational, incompressible and inviscid flows. I examined free surface flows under different types of forcing (moving pressure distribution and topography) where the Bond number was close to 1/3 and I obtained an equation for the graph of the free surface. I showed that there was a blow up in the linear analysis and I then went on to examine the weakly nonlinear extension. I studied this in the 2D and 3D cases and obtained a generalisation of the KP equation with a term which quantified the electric field interaction.

Currently the models that I have worked with have no charge on the interface, This isn't physically realistic. According to Maxwell's equations, the charge will migrate to the interface, but if the stress tensor is analysed, one finds that for the inviscid theory:

$$\left[\hat{\mathbf{n}} \cdot \Sigma \cdot \hat{\mathbf{t}}\right] = 0 \tag{1}$$

including a charge shows that [4]:

$$\left[\hat{\mathbf{n}} \cdot \Sigma \cdot \hat{\mathbf{t}}\right] = q\mathbf{E} \cdot \hat{\mathbf{t}} \tag{2}$$

So if charge on the interface is considered than it is obvious that one can only consider viscous flow. The goal of this analysis being to obtain a direct generalisation of the KdV equation which associated forcing and electric terms. One could alternatively perform analysis on the interface of two fluids as in the Benjamin-Ono equation as the equations are exactly the same, the Hilbert transform term modelling the contribution in the upper fluid in the Benjamin-Ono case or the electric field when there is no upper fluid. There are several ways that this problem can be analysed, one is to simply write down The Navier-Stokes equations and scale these thereby examining the equations in the same limit and look for a free surface that way [1]. In this paper a simple analysis was carried out using perturbation theory in the long wavelength and small amplitude approximation to obtain an expression for the Free surface.

Another method that was first covered in ([2]) was to decompose the velocity vector $\mathbf{u}$ as

$$\mathbf{u} = \nabla\varphi + \nabla \times \mathbf{A} \tag{3}$$

Then one uses a linear analysis to separate the two quantities [3] $(\varphi, \mathbf{A})$, where $\mathbf{A} = (0, \psi, 0)$ to:

$$\triangle\varphi = 0, \quad \frac{\partial\psi}{\partial t} = \nu\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) \tag{4}$$

In order to close the system of equations another equation must be added to the system, the conservation of charge which is applied as a boundary condition across the interface.[5]

$$\frac{\partial q}{\partial t} + \mathbf{u} \cdot \nabla_\eta q - q\hat{\mathbf{n}} \cdot [(\mathbf{u} \cdot \nabla)\mathbf{u}] + [\sigma \hat{\mathbf{n}} \cdot \mathbf{E}] \tag{5}$$

Where $\nabla_\eta$ is the covariant derivative on the free surface. Some authors have stated that the effect of viscosity is more important that the nonlinear effect in matching theory to experiment and that is where current analysis is centred.

The goal of the research will be the following:

1. Extend the analysis in [1] to include charge on the interface and look for travelling wave solutions.
2. Formulate electrohydrodynamics in terms of the decomposition (3) and look for linear solutions.
3. Add charge to the interface and expand on the model in point 2.

## References


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